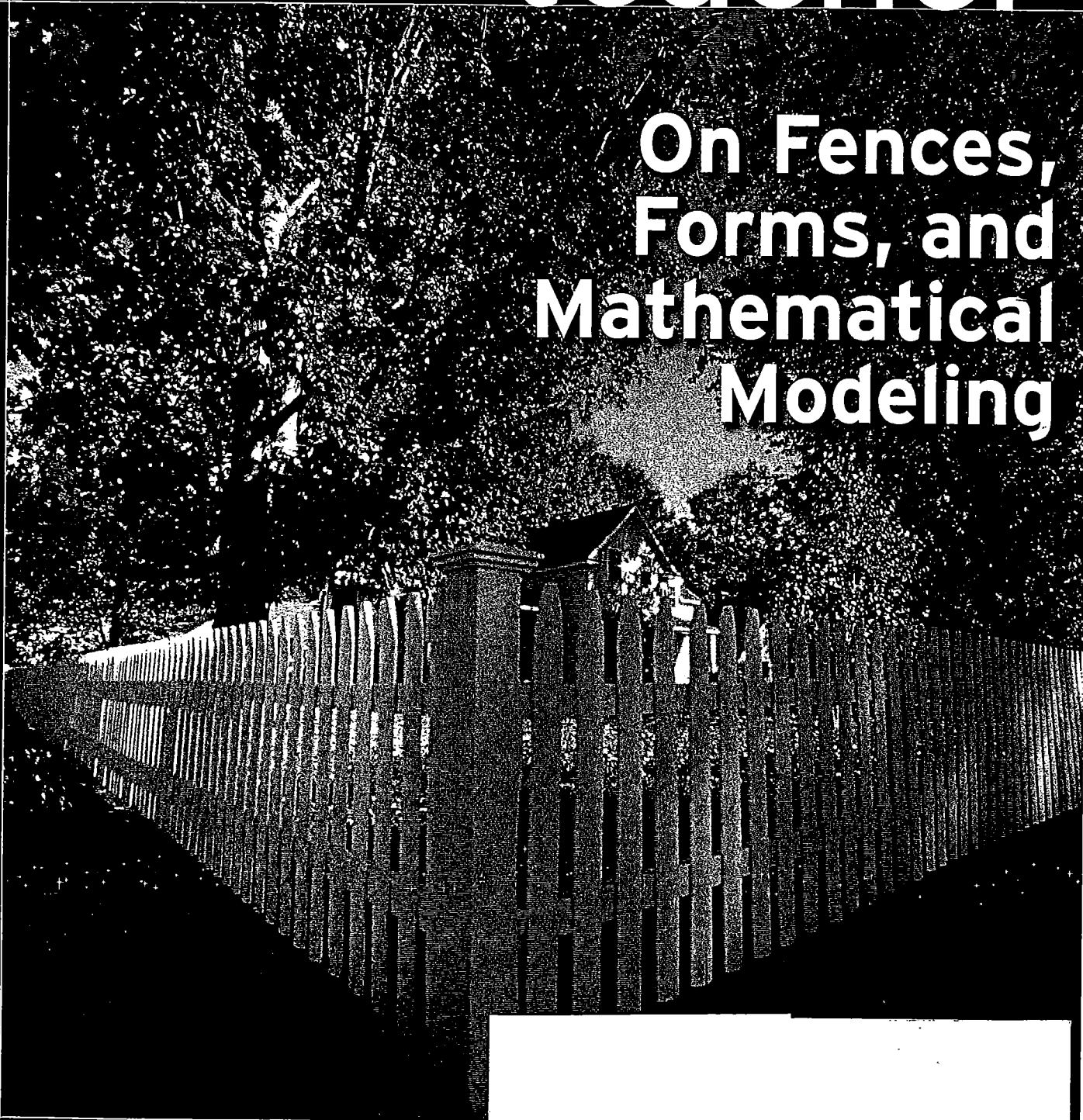


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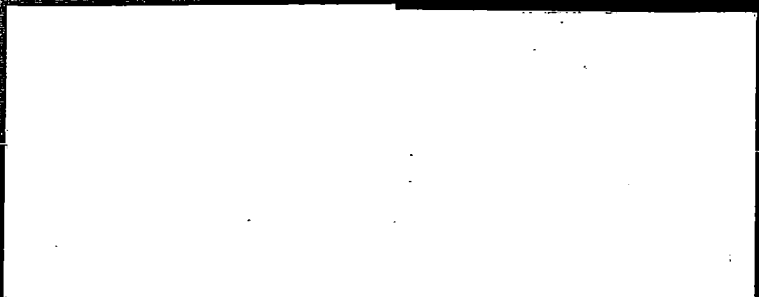
On Fences, Forms, and Mathematical Modeling



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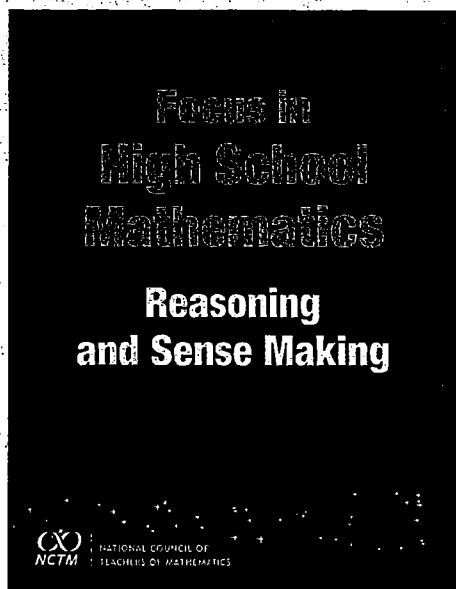


NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS



The NCTM High School Curriculum Project

Why It Matters to You



The message of *Focus in High School Mathematics: Reasoning and Sense Making*, NCTM's new (2009) publication on high school mathematics, is simple: Reasoning and sense making provide a focus for high school mathematics that will give students a foundation for their future success. This focus continues NCTM's emphasis on mathematical processes that stretches back to the central emphasis placed on problem solving in *An Agenda for Action* (NCTM 1980) and forward to the Process Standards of *Principles and Standards for School Mathematics* (NCTM 2000).

In fact, *Focus in High School Mathematics* argues that reasoning and sense making are foundational to all mathematical processes. For example, problem solving is impossible without reasoning, and sense making is inherent in making mathematical connections. A focus on reasoning and sense making places students' personal engagement in mathematical thinking at the forefront of high school mathematics.

While building on NCTM's continuing vision for school mathemat-

ics, *Focus in High School Mathematics* provides useful new perspectives that may help form a consensus around the changes that need to be made in high school mathematics. Moreover, the publication is poised to become the model for continued work by NCTM and others to improve high school mathematics.

WHY REASONING AND SENSE MAKING MATTER

All too many high school students graduate without the mathematical preparation they need to succeed in the future—as productive citizens who can make sound judgments based on mathematics (Programme for International Student Assessment 2007); as members of the workplace, ready to meet challenges we cannot imagine (Friedman 2007); and as potential members of a scientific and technical community, which is diminishing in the United States (Tapping America's Potential 2008). A focus on reasoning and sense making, coupled with important content, helps address these issues.

Learning how to execute a long list of mathematical procedures precisely may be less relevant in an age in which step-by-step solutions for many mathematical situations can be quickly found on the Internet (WolframAlpha 2009). To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results, as when students asked to simplify an expression somehow turn it into an equation to solve. Moreover, students may develop their own creative “rules,” such as “the square root of a sum is the sum of the square roots” (NCTM 2009, p. 12), which may sound correct but lack mathematical validity. This is not to suggest that learning procedures is irrelevant; *Focus in High School Mathematics* concludes, “Genuine procedural fluency requires both mastering technical skills and developing the understanding needed for using them appropriately” (p. 12).

Reasoning and sense making not only underlie students’ effective use of mathematics, they are also an important means through which students learn mathematics. Consider, for example, a student who learns the distance formula without understanding the mathematical reasoning on which it is based. When encountering a situation calling for the distance formula, the student may confuse it with other formulas and not remember whether the coordinates are added or subtracted, whether or not the sum or difference needs to be squared, and whether or not one needs to divide by 2. In contrast, a student who understands that the formula is really an application of the Pythagorean theorem will be more likely to reason out the relations underlying the formula and come to a correct conclusion. When students experience mathematics as a reasoning and sense-making enterprise, they are more likely to retain what they have learned, thus curtailing the seemingly unending cycle of reteaching prerequisite knowledge that is all too common in high school mathematics.

DEVELOPING REASONING HABITS

Focus in High School Mathematics takes the firm stand that reasoning and sense making should be a part of the high school mathematics classroom every day. To help teachers and others better understand how reasoning and sense making develop, a series of reasoning habits—a *reasoning habit* is defined as “a productive way of thinking that becomes common in the processes of mathematical inquiry and sense making” (p. 9)—are delineated. Rather than approaching these reasoning habits as a list of new topics to be taught, teach-

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ers should make them a routine and expected part of high school mathematics classrooms, extending across all mathematical topics and content areas.

These reasoning habits are divided into four broad categories—analyzing a problem, implementing a strategy, seeking and using connections, and reflecting on a solution to a problem. Although many of the more-specific reasoning habits, such as “seeking patterns and relationships” or “considering the reasonableness of a solution,” may seem familiar, they often receive limited attention in the classroom. Moreover, other reasoning habits may provide new insights into how reasoning and sense making may grow in the high school mathematics classroom.

For example, “looking for hidden structure” as a part of analyzing a problem suggests the need to look beyond the surface, as in adding an auxiliary line in a geometric situation or rewriting an algebraic expression into a form that reveals some important information that is not obvious, such as the maximum value of the expression. In reflecting on a solution to a problem, students might “reconcile different approaches,” including those proposed by their classmates. This reasoning habit may help students recognize

which approaches are most useful in various contexts and may also build a deeper appreciation for “elegant” proofs that provide insight into why a particular result may be true.

REASONING AND SENSE MAKING IN THE CURRICULUM

Many groups have developed lists of topics that should be included in specific course sequences (Achieve 2007a, 2007b; ACT 2007; The College Board 2006, 2007), and the “Common Core Standards” effort (McNeil 2009) has the goal of producing shared standards for mathematics across forty-six states and

three territories. Although developing more consistency in what is taught may well have benefits, *Focus in High School Mathematics* argues for a different kind of consistency in ensuring the foundational importance of reasoning and sense making. Whatever topics are taught, students must learn them in a way that deepens their mathematical thinking and reasoning.

Focus in High School Mathematics takes the position that NCTM has already set forth the mathematical content that high school students should experience in *Principles and Standards for School Mathematics* (NCTM 2000). However, the new publication provides useful insights into how reasoning and sense making can be infused into five strands of the high school mathematics curriculum—reasoning with number and measurement, reasoning with algebraic symbols, reasoning with functions, reasoning with geometry, and reasoning with probability and statistics. Each of these strands includes a number of key elements, which provide a broad structure for thinking about how to focus that strand on reasoning and sense making. For example, reasoning with algebraic symbols suggests that the curriculum be focused on the following:

- Meaningful use of symbols
- Mindful manipulation
- Reasoned solving
- Connections between algebra and geometry
- Connections between expressions and functions

Each strand is described in a separate chapter of section 2. Each chapter provides a number of examples illustrating how reasoning and sense making might be promoted within that strand and also how they might support learning of that strand.

HOW TO CHANGE THE FOCUS OF HIGH SCHOOL MATHEMATICS

Refocusing high school mathematics on reasoning and sense making is a formidable task and will require intensive and sustained effort. Section 3 of *Focus in High School Mathematics* directly addresses issues related to this endeavor. One central issue is the different experiences students have in high school mathematics, where students enrolled in “advanced” or “honors” classes often have many more opportunities to engage in reasoning and sense-making activities than those who are in “low” or “regular” classes. Moreover, placement of students in these lower-level courses can too often be predicted by demographic variables, such as race, ethnicity, or socioeconomic status. *Focus in High School Mathematics* argues that “mathematical reasoning and sense making must be evident in the mathematical experiences of all students” (p. 93).

The publication also addresses the need for coherence, including the importance of instructional practices and tasks that promote reasoning and sense making, the need for vertical alignment both with pre- and post-high school mathematics, and the impact of assessment, both formative and summative. The final chapter begins with the proposition that “everyone involved must work together to ensure that reasoning and sense making are central foci of high school mathematics programs” (p. 107). A set of questions is posed to provoke discussion among those with a stake in high school mathematics.

In some sense, *Focus in High School Mathematics* may be considered a conversation starter or perhaps a conversa-

tion extender. The publication by itself will achieve little unless all involved in high school mathematics take up the challenge. NCTM will do its part by producing a series of topic books that will closely follow publication of *Focus in High School Mathematics* and will provide details on how we can meet the challenge. Three volumes will address practical means to promote reasoning and sense making within the curricular areas of algebra, geometry, and probability and statistics. An additional volume will discuss ways in which we can ensure that all high school students receive an equitable mathematics preparation, whatever their background or perceived ability level, and additional volumes will provide further guidance on other central issues. Professional development and other activities are being planned, and NCTM is making a long-term commitment to addressing high school mathematics.

In the end, however, NCTM can only hope to encourage the changes that are needed. All stakeholders must join forces and work together in meaningful ways to ensure that five years from now—never mind thirty years from now—we will not still be recounting the story of missed opportunities to significantly improve high school mathematics across the United States. We simply cannot afford to wait any longer to address the large-scale changes that are needed. The success of our students and of our nation depends on addressing them now.

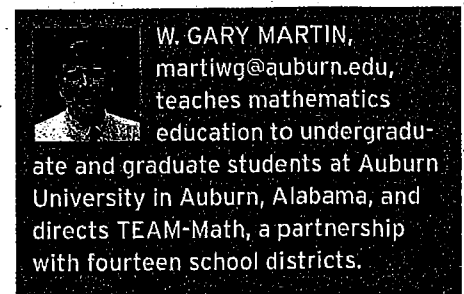
It is our hope that *Focus in High School Mathematics* will prove a useful tool in achieving this goal, as teachers begin discussions around the publication with one another and with others involved in high school mathematics.

Editor’s note: W. Gary Martin was the chair of both the writing group for *Focus in High School Mathematics: Reasoning and Sense Making* and the planning committee for NCTM’s High School Curriculum Project.

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